Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Friday 24 May 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 10 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

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2. (a) Use the binomial theorem to find all the terms of the expansion of

$$(2 + 3x)^4.$$

Give each term in its simplest form.

(4)

(b) Write down the expansion of

$$(2 - 3x)^4$$

in ascending powers of x, giving each term in its simplest form.

(1)

---

3. $$\mathrm{f}(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $\mathrm{f}(x)$,

(a) show that $a = -9$,

(2)

(b) factorise $\mathrm{f}(x)$ completely.

(4)

Given that

$$\mathrm{g}(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^{y}) + 18,$$

(c) find the values of y that satisfy $\mathrm{g}(y) = 0$, giving your answers to 2 decimal places where appropriate.

(3)

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4. $$y = \frac{5}{(x^2+1)}.$$

(*a*) Copy and complete the table below, giving the missing value of *y* to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

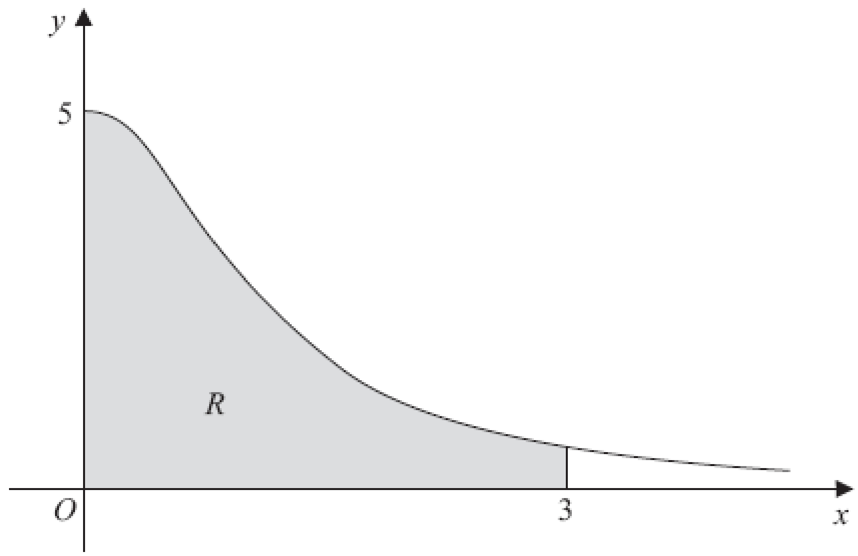

Figure 1

Figure 1 shows the region *R* which is bounded by the curve with equation $y = \frac{5}{(x^2+1)}$, the *x*-axis and the lines $x = 0$ and $x = 3$.

(*b*) Use the trapezium rule, with all the values of *y* from your table, to find an approximate value for the area of *R*.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^3 4 + \frac{5}{(x^2+1)}\ \mathrm{d}x,$$

giving your answer to 2 decimal places.

(2)

5.

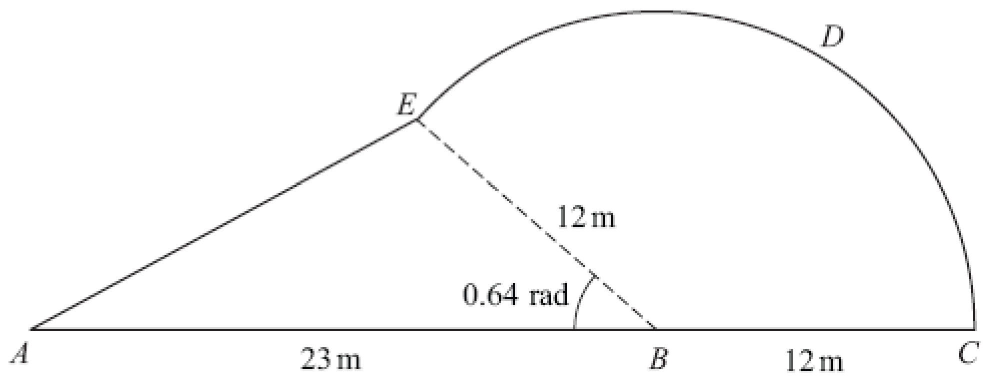

Figure 2

Figure 2 shows a plan view of a garden.

The plan of the garden *ABCDEA* consists of a triangle *ABE* joined to a sector *BCDE* of a circle with radius 12 m and centre *B*.

The points *A*, *B* and *C* lie on a straight line with *AB* = 23 m and *BC* = 12 m.

Given that the size of angle *ABE* is exactly 0.64 radians, find

(*a*) the area of the garden, giving your answer in m^2, to 1 decimal place,

(4)

(*b*) the perimeter of the garden, giving your answer in metres, to 1 decimal place.

(5)

6.

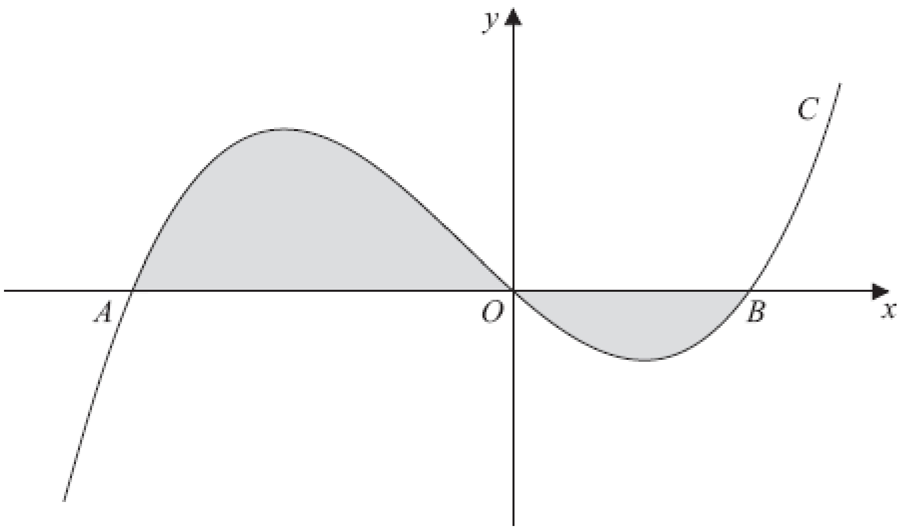

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2).$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

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7. (i) Find the exact value of x for which

$$\log_2 (2x) = \log_2 (5x + 4) - 3.$$

(4)

(ii) Given that

$$\log_a y + 3 \log_a 2 = 5,$$

express y in terms of a.

Give your answer in its simplest form.

(3)

---

8. (i) Solve, for $-180° \leq x < 180°$,

$$\tan(x - 40°) = 1.5,$$

giving your answers to 1 decimal place.

(3)

(ii) (*a*) Show that the equation

$$\sin\theta \tan\theta = 3\cos\theta + 2$$

can be written in the form

$$4\cos^2\theta + 2\cos\theta - 1 = 0.$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\sin\theta \tan\theta = 3\cos\theta + 2,$$

showing each stage of your working.

(5)

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9. The curve with equation

$$y = x^2 - 32\sqrt{x} + 20, \qquad x > 0,$$

has a stationary point P.

Use calculus

(*a*) to find the coordinates of P,

(6)

(*b*) to determine the nature of the stationary point P.

(3)

10.

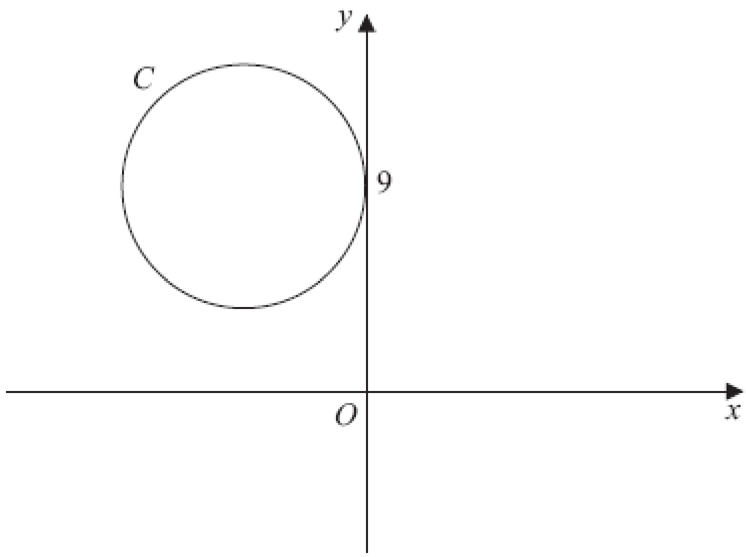

Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T.

(b) Find the length of PT.

(3)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\{r=\}\ \frac{2}{3}$	B1
		(1)
(b)	$\{p=\}\ 8$	B1 cao
		(1)
(c)	$\{S_{15}=\}\ \frac{18\left(1-(\frac{2}{3})^{15}\right)}{1-\frac{2}{3}}$	M1
	$\{S_{15}=53.87668...\} \Rightarrow S_{15}=\text{awrt } 53.877$	A1
		(2)
		[4]

	Notes for Question 1
(a)	B1: Accept $\frac{12}{18}$, $0.\dot{6}$ or 0.6 recurring, or even 0.667 (3sf) but not 0.6 or 0.67
(b)	B1: accept 8 only
(c)	M1: Applies this formula $S_{15}=\frac{18\left(1-(\text{their } r)^{15}\right)}{1-(\text{their } r)}$, can be implied by their answer. For this mark they may use any value for r except $r=1$ or $r=0$ (even 3/2 or -6 may be used) A1: Answers which round to 53.877
Alternative method for (c)	M1: (Adding terms is an unlikely method for this question) Need to see 15 terms listed as 18+12+……0.06165877 or can be implied by correct answer A1: awrt 53.877 **Answer only** : 53.9 is M0A0 with no working, but 53.877 with no working is M1A1

Question Number	Scheme	Marks
2. (a)	$(2+3x)^4$ **-** Mark (a) and (b) together $2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$ First term of 16 $\left({}^4C_1 \times \ldots \times x\right) + \left({}^4C_2 \times \ldots \times x^2\right) + \left({}^4C_3 \times \ldots \times x^3\right) + \left({}^4C_4 \times \ldots \times x^4\right)$ $= (16+)\ 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0	 B1 M1 A1 A1 **(4)**
(b)	$(2-3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$	B1ft **(1)** **5**
Alternative method (a)	$(2+3x)^4 = 2^4(1+\frac{3x}{2})^4$ $2^4(1 + {}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$ Scheme is applied exactly as before	
	Notes for Question 2	
(a) (b)	B1: The constant term should be 16 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of *x*. Accept ${}^4C_1$ or $\binom{4}{1}$ or 4 **as a coefficient, and** ${}^4C_2$ or $\binom{4}{2}$ or 6 **as another……..** Pascal's triangle may be used to establish coefficients. A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method. A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines) B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five terms** ft and must be **subtracting the x and x^3 terms** Allow terms in (b) to be in descending order and allow +-96*x* and +-216x^3 in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
	e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $= (16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5 **Fully correct answer with no working can score B1 in part (a) and B1 in part (b).** The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned. **Squaring the bracket and squaring again** may also earn **B1 M0 A0 A0 B1 if correct** **Omitting the final term** but otherwise correct is B1 M1 A1 A0 B0ft so 3/5 If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)	

Question Number	Scheme	Marks
3. (a)	Either (Way 1) : Attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ $\mathrm{f}(3) = 54 - 45 + 3a + 18 = 0 \Rightarrow 3a = -27 \Rightarrow a = -9$ * Or (Way 2): Assume a = -9 and attempt $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ f(3) = 0 so (x -3) is factor	M1 A1 * cso **(2)**
	Or (Way 3): $\left(2x^3 - 5x^2 + ax + 18\right) \div \left(x - 3\right) = 2x^2 + px + q$ where p is a number and q is an expression in terms of a Sets the remainder 18+3a+9 = 0 and solves to give $a = -9$	M1 A1* cso **(2)**
(b)	Either (Way 1): $\mathrm{f}(x) = (x - 3)(2x^2 + x - 6)$ $= (x - 3)(2x - 3)(x + 2)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain x = -2 **or** x = 3/2 Uses trial or factor theorem to obtain both x = -2 **and** x = 3/2 Puts three factors together (see notes below) Correct factorisation : $(x - 3)(2x - 3)(x + 2)$ or $(3 - x)(3 - 2x)(x + 2)$ or $2(x - 3)(x - \frac{3}{2})(x + 2)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x - 3)(2x - 3)(x + 2)$ otherwise need working	M1A1M1A1
(c)	$\{3^y = 3 \Rightarrow\}$ $\underline{y = 1}$ or g(1) = 0 $\{3^y = 1.5 \Rightarrow\} \log\left(3^y\right) = \log 1.5$ or $y = \log_3 1.5$ $\{y = 0.3690702...\} \Rightarrow y =$ awrt 0.37	B1 M1 A1 **(3)** **[9]**
	Notes for Question 3	
(a)	M1 for attempting either $\mathrm{f}(3)$ or $\mathrm{f}(-3)$ – with **numbers substituted into expression** A1 for applying $\mathrm{f}(3)$ **correctly**, setting the result **equal to 0**, and manipulating this correctly to give the result given on the paper i.e. $a = -9$. (Do not accept $\boldsymbol{x}$ = -9) Note that the answer is given in part (a). If they **assume** a = -9 and **verify** by factor theorem or division they must state (x – 3) **is a factor** for A1 (or equivalent such as QED or a tick).	
(b)	1st M1: attempting to divide by $(x - 3)$ leading to a 3TQ beginning with the correct term, usually $2x^2$. (Could divide by $(3 - x)$, in which case the quadratic would begin $-2x^2$.) This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. 1st A1: usually for $2x^2 + x - 6$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) NB: $(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M0A0, $(x - 3)(x - \frac{3}{2})(2x + 4)$ is M1A1M1A0, but $2(x - 3)(x - \frac{3}{2})(x + 2)$ is M1A1M1A1.	
(c)	B1: $\underline{y = 1}$ seen as a solution – may be spotted as answer – no working needed. Allow also for g(1) = 0. M1: Attempt to take logs to solve $3^y = \alpha$ or even $3^{ky} = \alpha$, but not $6^y = \alpha$ where $\alpha > 0$ and $\alpha \neq 3$ & was a root of f(x) = 0 (ft their factorization) A1: for an answer that rounds to 0.37. If a third answer is included (and not “rejected”) such as ln(-2) lose final A mark	

Question Number	Scheme	Marks
4.		x | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 y | 5 | 4 | 2.5 | 1.538 | 1 | 0.690 | 0.5	
(a)	$\{\text{At } x = 1.5,\}\ y = 1.538$ (only)	B1 **cao**
		[1]
(b)	$\frac{1}{2} \times 0.5$;	B1 oe
	$\underline{\{5 + 0.5 + 2(4 + 2.5 + \text{ their } 1.538 + 1 + 0.690)\}}$ For structure of $\underline{\{................\}}$;	M1<u>A1ft</u>
	$\frac{1}{2} \times 0.5 \times \underline{\{(5 + 0.5) + 2(4 + 2.5 + \text{ their } 1.538 + 1 + 0.690)\}}\ \{= \frac{1}{4}(24.956) = 6.239\} = \text{awrt } 6.24$	A1
		[4]
(c)	Adds Area of Rectangle or first integral $= 3 \times 4$ or $[4x]_0^3$ to **previous answer**	M1
	So required estimate = $\{"6.239" + 12 = "18.239"\} = "\text{awrt } 18.24"$ (or 12 + previous answer).	A1ft
	N.B. $7 \times 4 +$ previous answer is M0A0 (added 4 seven times because 7 numbers in table)	**[2]**
		7

	Notes for Question 4
(a)	B1: 1.538
(b)	B1: for using $\frac{1}{2} \times 0.5$ or $\frac{1}{4}$ or equivalent. M1: requires the correct $\{......\}$ bracket structure. It needs the first bracket to contain first y value **plus** last y value and the second bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from 2nd bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values A1ft: for the correct bracket $\{......\}$ following through candidate's y value found in part (a). A1: for answer which rounds to 6.24. NB: Separate trapezia may be used : B1 for 0.25, M1 for 1/2 $h(a + b)$ used 5 or 6 times (and A1ft if it is all correct) Then A1 as before. Special case: Bracketing mistake $0.25 \times (5 + 0.5) + 2(4 + 2.5 + \text{ their } 1.538 + 1 + 0.690)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). An answer of 20.831 usually indicates this error.
(c)	M1: Relates **previous answer (not integral of previous answer**) to this question by integrating 4 between limits, and adding, or by using geometry to find rectangle and adding. A1ft: for 12 + answer to (b)
Alternative method (c)	Those who do a trapezium rule for part (b)- using the table from (a) with 4 added to each cell of the table Get: M1 for $"\textit{their}\ \frac{1}{4}" \times \underline{\{9 + 4.5 + 2(8 + 6.5 + \text{ their } 5.538 + 5 + 4.690)\}}$ = (structure must be correct – allow one copying error only) And A1ft: for awrt 18.24 (or 12 + previous answer).

Question Number	Scheme	Marks
5. (a)	Mark (a) and (b) together.	
	Usually answered in radians: Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{1}{2}(12)^2(\text{angle})$ or both	M1
	Area $= \frac{1}{2}(23)(12)\sin 0.64$ **or** $\frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091...$ **or** $180.1146711...\}$	A1
	Area $= \frac{1}{2}(23)(12)\sin 0.64 + \frac{1}{2}(12)^2(\pi - 0.64)$ $\{= 82.41297091... + 180.1146711...\}$	A1
	$\{\text{Area} = 262.527642...\}$ = awrt 262.5 (m^2) or $262.4(m^2)$ or 262.6 (m^2)	A1
		(4)
(b)	$CDE = 12 \times (angle)$, $= 12(\pi - 0.64)\{\Rightarrow CDE = 30.01911...\}$	M1, A1
	$AE^2 = 23^2 + 12^2 - 2(23)(12)\cos(0.64) \Rightarrow AE^2 =$ or $AE =$ $\{AE = 15.17376...\}$	M1
	Perimeter $= 23 + 12 + 15.17376... + 30.01911...$	M1
	$= 80.19287... =$ awrt 80.2 (m)	A1
		(5)
		[9]

	Notes for Question 5
(a)	M1: uses either area of triangle formula as given in mark scheme, or area of sector or both (may be implied by answer)
	A1: one correct area expression (with **correct angle** used) $\frac{1}{2}(23)(12)\sin 0.64$ or $\frac{1}{2}(12)^2(\pi - 0.64)$ or see awrt 82.4 **or** awrt 180 (180 may be split as 226.2(semicircle) minus 46.1(small sector))
	A1: two correct area expressions (with correct angles) **added together (**allow 2.5 as implying $\pi - 0.64$) or see awrt 82.4 **+** awrt 180 (or 226 - 46)
	A1: answers which round to 262.5 or 262.4 or 262.6
(b)	1st M1 for attempt to use $s = r\theta$ (any angle)
	1st A1 for $\pi - 0.64$ in the formula (or 2.5)
	2nd M1: Uses correct cosine rule to obtain AE or AE^2(this may appear in part (a))
	3rd M1(**independent)**: Perimeter = $23 + 12 +$ their AE + their CDE
	2nd A1: awrt 80.2 (ignore units – even incorrect units)
Degrees (a)	Uses either $\frac{1}{2}ab\sin(\text{angle})$ **or** $\frac{\text{angle in degrees}}{360} \times \pi(12)^2$ or both for M1
	Area $= \frac{1}{2}(23)(12)\sin 36.7$ **or** $\frac{(180\text{-}36.7)}{360} \times \pi(12)^2$ $\{= \textit{awrt}\ 82.4...$ **or** $180\}$ A1
	Area $= \frac{1}{2}(23)(12)\sin 36.7 + \frac{(180\text{-}36.7)}{360} \times \pi(12)^2$ $\{= \textit{awrt}\ 82.4... + 180\}$ A1
	Final mark as before
(b)	$CDE = \frac{\text{Angle in degrees}}{360} \times 24\pi$, $= \frac{180 - 36.7}{360} \times 24\pi\{\Rightarrow CDE = 30.01268...\}$ M1, A1
	Final three marks as before

Question Number	Scheme	Marks
6. (a)	Seeing -4 and 2.	B1 **(1)**
(b)	$x(x+4)(x-2) = \underline{x^3 + 2x^2 - 8x}$ or $\underline{x^3 - 2x^2 + 4x^2 - 8x}$ (without simplifying)	<u>B1</u>
	$\int (x^3 + 2x^2 - 8x)\,dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2} \{+c\}$ or $\frac{x^4}{4} - \frac{2x^3}{3} + \frac{4x^3}{3} - \frac{8x^2}{2} \{+c\}$	M1A1ft
	$\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{0} = (0) - \left(64 - \frac{128}{3} - 64\right)$ **or** $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{0}^{2} = \left(4 + \frac{16}{3} - 16\right) - (0)$	dM1
	One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7) **or** other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7)	A1
	Hence Area = "*their* $42\frac{2}{3}$" + "*their* $6\frac{2}{3}$" or Area = "*their* $42\frac{2}{3}$" − "− *their* $6\frac{2}{3}$"	dM1
	$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)	A1
	(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)	**(7)** **[8]**
	Notes for Question 6	
(a)	B1: Need both –4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or A = -4, B = 2 or indeed any indication of -4 and 2 – check graph also	
(b)	B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here) M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms A1ft: completely correct integral **following through** from their CUBIC expansion (if only quadratic or quartic this is A0) dM1: (dependent on previous M) substituting EITHER -a and 0 and subtracting either way round OR similarly for 0 and b. **If their limits –*a* and *b* are used in ONE integral, apply the Special Case below**. A1: Obtain **either** $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)*from the integral from -4 to 0* **or** $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) *from the integral from 0 to 2;* NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ **or** $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0. dM1 (depends on first method mark) **Correct** method **to obtain shaded area** so adds two positive numbers (areas) together or uses their **positive** value minus **their negative** value, **obtained from** two separate **definite integrals**. A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with **no errors** seen. For full marks on this question there must be **two** definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen. **(**Trapezium rule **gets no marks after first two B marks)**	
(b)	**Special Case: one integral only from –*a* to *b*:** B1M1A1 available as before, then $\left[\frac{x^4}{4} + \frac{2x^3}{3} - \frac{8x^2}{2}\right]_{-4}^{2} = (4 + \frac{16}{3} - 16) - \left(64 - \frac{128}{3} - 64\right) = -6\frac{2}{3} + 42\frac{2}{3}$ **=….** **dM1** for correct use of their limits –a and b and **subtracting** either way round**.** A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)	

Question Number	Scheme	Marks
7. (i) Method 1	$\log_2\left(\frac{2x}{5x+4}\right) = -3$ or $\log_2\left(\frac{5x+4}{2x}\right) = 3$, or $\log_2\left(\frac{5x+4}{x}\right) = 4$ (see special case 2)	M1
	$\left(\frac{2x}{5x+4}\right) = 2^{-3}$ or $\left(\frac{5x+4}{2x}\right) = 2^3$ or $\left(\frac{5x+4}{x}\right) = 2^4$ or $\left(\log_2\left(\frac{2x}{5x+4}\right)\right) = \log_2\left(\frac{1}{8}\right)$	M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on previous Ms and must be this equation or equivalent)	dM1
	$x = \frac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work	A1 **cso** **(4)**
7(i) Method 2	$\log_2(2x) + 3 = \log_2(5x+4)$	
	So $\log_2(2x) + \log_2(8) = \log_2(5x+4)$ (3 replaced by $\log_2 8$)	2nd M1
	Then $\log_2(16x) = \log_2(5x+4)$ (addition law of logs)	1st M1
	Then final M1 A1 as before	dM1A1
(ii)	$\log_a y + \log_a 2^3 = 5$	M1
	$\log_a 8y = 5$ — Applies product law of logarithms.	dM1
	$y = \frac{1}{8}a^5$ — $y = \frac{1}{8}a^5$	A1cao
		(3) **[7]**
	Notes for Question 7	
(i)	1st M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms in *x* into one** log term in *x* 2nd M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\frac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an earlier error. **Use of** 3^2 **is M0** 3rd dM1: Obtains **correct** linear equation in *x*. usually the one in the scheme and attempts $x =$ A1: cso Answer of 4/11 with **no** suspect log work preceding this.	
(ii)	M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ *or* $\log_a 8$ dM1: (should not be following M0) Uses addition law of logs to give $\log_a 2^3 y = 5$ or $\log_a 8y = 5$	
(i)	**Special case 1:** $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ or $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \frac{\log_2(2x)}{\log_2(5x+4)} = -3 \Rightarrow \log_2 \frac{2x}{5x+4} = -3 \Rightarrow \frac{2x}{5x+4} = 2^{-3} \Rightarrow x = \frac{4}{11}$ each attempt scores M0M1M1A0 – special case	
	Special case 2: $\log_2(2x) = \log_2(5x+4) - 3 \Rightarrow \log_2 2 + \log_2 x = \log_2(5x+4) - 3$, is M0 until the two log terms are combined to give $\log_2\left(\frac{5x+4}{x}\right) = 3 + \log_2 2$. This earns M1 Then $\left(\frac{5x+4}{x}\right) = 2^4$ or $\log_2\left(\frac{5x+4}{x}\right) = \log_2 2^4$ gets second M1. Then scheme as before.	

Question Number	Scheme	Marks
8. (i)	$(\lvert\alpha\rvert = 56.3099...)$	
	$x = \{\alpha + 40 = 96.309993...\}$ = **awrt 96.3**	B1
	$x - 40^\circ = -180 + "56.3099"...$ or $x - 40^\circ = -\pi + "0.983"...$	M1
	$x = \{-180 + 56.3099... + 40 = -83.6901...\}$ = **awrt -83.7**	A1
		(3)
(ii)(a)	$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	M1
	$\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	dM1
	$1-\cos^2\theta = 3\cos^2\theta + 2\cos\theta \Rightarrow 0 = 4\cos^2\theta + 2\cos\theta - 1$ *	A1 cso *
		(3)
(b)	$\cos\theta = \dfrac{-2 \pm \sqrt{4-4(4)(-1)}}{8}$	M1
	or $4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0$, or $(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0$, $q \neq 0$ so $\cos\theta = ...$	
	One solution is 72° or 144°, Two solutions are 72° and 144°	A1, A1
	$\theta = \{72, 144, 216, 288\}$	M1 A1
		(5) **[11]**

	Notes for Question 8
(i)	B1: 96.3 by any or no method M1: Takes 180 degrees from arctan (1.5) or from their "96.3" May be implied by A1. (Could be obtained by adding 180, then subtracting 360). A1: awrt –83.7 Extra answers: ignore extra answers outside range. Any extra answers in range lose final A mark (if earned) Working in radians – could earn M1 for $x - 40^\circ = -\pi + "0.983"...$ so B0M1A0
(ii) (a)	M1: uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or equivalent in equation (not just $\tan = \dfrac{\sin}{\cos}$, with no argument) dM1: uses $\sin^2\theta = 1-\cos^2\theta$ (quoted correctly) in equation A1: completes proof correctly, with no errors to give printed answer*. Need at least three steps in proof and need to achieve the correct quadratic with all terms on one side and "=0"
(b)	M1: Attempts to solve quadratic by correct quadratic formula, or completion of the square . Factorisation attempts score M0. 1st A1: Either 72 **or** 144, 2nd A1: both 72 **and** 144 (allow 72.0 etc.) M1: 360 – "a previous solution" (provided that cos was being used) (not dependent on previous M) A1: All four solutions correct (Extra solutions in range lose this A mark, but outside range - ignore) (**Premature approximation**: e.g. 71.9, 144.1, 288.1 and 215.9 – lose first A1 then ft other angles) Do **not** require degrees symbol for the marks **Special case: Working in radians** M1: as before, A1 for either $\theta = \frac{2}{5}\pi$ or $\theta = \frac{4}{5}\pi$ or decimal equivalents, and 2nd A1: both M1: $2\pi - \alpha_1$ or $2\pi - \alpha_2$ then A0 so 4/5

Question Number	Scheme	Marks
9. (a)	$\left\{\frac{dy}{dx}=\right\}\ 2x-16x^{-\frac{1}{2}}$	M1 A1
	$2x-16x^{-\frac{1}{2}}=0 \Rightarrow\ x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$,or $2x-=16x^{-\frac{1}{2}}$ then squared then obtain $x^3=$ [or $2x-16x^{-\frac{1}{2}}=0 \Rightarrow x=4$ (no wrong work seen)]	M1
	$(x^{\frac{3}{2}}=8 \Rightarrow)\ x=\ 4$	A1
	$x=4,\ y=\ 4^2-32\sqrt{4}+20=-28$ (ignore y = 100 as second answer)	M1 A1
		(6)
(b)	$\left\{\frac{d^2y}{dx^2}=\right\}\ 2+8x^{-\frac{3}{2}}$	M1 A1
	$(\frac{d^2y}{dx^2}>0 \Rightarrow)\ y$ is a minimum (there should be no wrong reasoning)	A1
		(3) **[9]**
(b)	<u>**Alternative Method: Gradient Test:**</u> M1 for finding the gradient either side of their *x*-value from part (a). A1 for <u>both gradients calculated correctly to 1 significant figure</u>, then <u>using < 0 and > 0 respectively *maybe by use of sketch or table*</u>. (See appendix for gradient values. This is **not ft their *x***) A1 states minimum needs M1A1 to have been awarded.	
	Notes for Question 9	
(a)	1st M1: At least one term differentiated correctly, so $x^2 \to 2x$, or $32\sqrt{x} \to 16x^{-\frac{1}{2}}$, or $20 \to 0$ A1: This answer or equivalent e.g. $2x-\frac{16}{\sqrt{x}}$ 2nd M1: Sets their $\frac{dy}{dx}$ to 0, and solves to give $x^{\frac{3}{2}}=,\ x^{-\frac{3}{2}}=$ *or* $x^3=$ after correct squaring or spots x = 4 (NB $\left\{\frac{d^2y}{dx^2}=0\right\}$ *so* $2+8x^{-\frac{3}{2}}=0$ **is M0)** N.B. Common error: Putting derivative = 0 and merely obtaining x = 0 is M0A0, then M0A0 for next two marks. (The first two marks in (a) and marks for second derivative may be earned in part (b).) A1: x = 4 cao [x = -4 is A0 and $x=\pm4$ is also A0] 3rd M1: Substitutes **their positive** found x **(NOT zero)** into $y=x^2-32\sqrt{x}+20,\ x>0$.**Should follow attempting to set** $\frac{dy}{dx}$ **= 0 and not setting** $\frac{d^2y}{dx^2}=0$ A1: -28 cao (Does not need to be written as coordinates)	
(b)	M1: Attempts differentiation of their first derivative with at least one term differentiated correctly. Should be seen or referred to (in part (b)) in determining the nature of the stationary point. A1: Answer in scheme or equivalent A1: States minimum (Second derivative should be correct- can follow incorrect positive *x*. Needs M1A1 to have been awarded- should not follow incorrect reasoning – (need not say $\frac{d^2y}{dx^2}>0$ but should not have said $\frac{d^2y}{dx^2}=0$ for example)	

Question Number	Scheme	Marks
10. (a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$	M1
	Equation of form $(x - a)^2 + (y - b)^2 = 5^2$, with values for a and b	M1
	$(x + 5)^2 + (y - 9)^2 = 25 = 5^2$	A1
		(3)
	$P(8, -7)$. Let centre of circle $= X(-5, 9)$	
(b)	$PX^2 = \left(8 - "-5"\right)^2 + \left(-7 - "9"\right)^2$ or $PX = \sqrt{(8 - -5)^2 + (-7 - 9)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = \left(PX\right)^2 - 5^2$ with numerical PX	dM1
	$PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	A1 **cso**
		(3) **[6]**
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$	M1
	Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes (0, 9) giving $+9^2 \pm 2b \times 9 + c = 0$	M1
	$x^2 + y^2 + 10x - 18y + 81 = 0$	A1
		(3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first)	M1
	M1: Use either of the correct points with P (8, -7) and distance between two points formula	dM1
	A1: 20	A1cso
		(3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$	M1
	Square roots to give $PT \left\{= \sqrt{400}\right\} = 20$	dM1A1
		(3)
	Notes for Question 10	
	The three marks in (a) each require a circle equation – (see special cases which are not circles)	
(a)	M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k > 0$ or a positive value)	
	M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2	
	A1: correct circle equation in any equivalent form	
	Special cases $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ is **not a circle** equation so M0M0A0	
	Also $(x \pm 5)^2 + (y - 9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0	
	But $(x - 0)^2 + (y - 9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X , as centre was not labelled in the question.	
	N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0.	
	dM1**:** Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to *P***) or uses appropriate complete method involving trigonometry	
	A1: 20 cso	

Question Number	Scheme	Marks
Aliter **9. (b)** ***Way 2***	**Gradient Test Method:** $\frac{dy}{dx} = 2x - 16x^{-\frac{1}{2}}$ ***Helpful table!***	

x	$\frac{dy}{dx}$
3	-3.2376
3.1	-2.88739
3.2	-2.54427
3.3	-2.20771
3.4	-1.87722
3.5	-1.55236
3.6	-1.23274
3.7	-0.918
3.8	-0.60783
3.9	-0.30191
4	0
4.1	0.298163
4.2	0.592799
4.3	0.884115
4.4	1.172299
4.5	1.457528
4.6	1.739962
4.7	2.01975
4.8	2.297033
4.9	2.571937
5	2.844582